**Question 1 [1p.]** The slope of the tangent to the graph of the function

$$f(x) = -x^3 + 3x^2 - 7x - 2$$

at the point $P = (1, -5)$ is equal to

    **A.** $-4$    **B.** $4$    **C.** $-2$    **D.** $2$

**Question 2 [1p.]** The equation $\cos^2 x = \cos x$ in the interval $[-\pi, 0]$

    **A.** has exactly one solution.        **B.** has exactly two solutions.

    **C.** has exactly three solutions.     **D.** has no solutions.

**Question 3 [1p.]** The expression

$$\frac{1}{a - b} - \frac{1}{\left(\frac{a}{b} - 1\right)a - \left(\frac{b}{a} - 1\right)b},$$

for $a, b \neq 0$ and $|a| \neq |b|$, is equal to

    **A.** $\frac{a^4 + b^4}{a^3 - b^3}$    **B.** $\frac{a^3 + b^3}{a^4 - b^4}$    **C.** $\frac{a^3 + b^3}{a^2 - b^2}$    **D.** $\frac{a^4 + b^4}{a^4 - b^4}$.

**Question 4 [1p.]** The limit

$$\lim_{n \to \infty} \frac{-7n^3 + n^2 + 2n}{n^3 + 3n^2 + 4n^5}$$

is equal to

    **A.** $-\frac{7}{4}$    **B.** $-7$    **C.** $0$    **D.** $-\infty$

**Question 5 [2p.]** Assume that

$$f(x) = \frac{ax + b}{cx + d},$$

where $b = 11$, $d = 12$ and $cx + d \neq 0$. The point $A = (6, -4)$ is the center of symmetry of the graph of the function $f$. Compute the ratio $\frac{b}{a}$. Enter the first three digits of the decimal expansion of the result.

    ANSWER:   | | | |

**Question 6 [3p.]** Prove that for all real numbers $a$, $b$, such that $4a^2 + b^2 \leqslant 4$ the inequality $2a + b \leqslant 3$ holds true.

**Question 7 [3p.]** For what values of the parameter $k$ the domain of the function

$$f(x) = \sqrt{(k + 3)x^2 + (k + 3)x + 2}$$

    is the set of real numbers?

**Question 8 [3p.]** Consider the numbers $a = \log 8 + 3 \log 5$ and $b = \frac{\log_9 7}{\log_7 49}$. Calculate $a^b$.

**Question 9 [4p.]** We randomly create the nine-digit number with different digits taken from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Calculate the probability of getting the odd number where the digits 5 and 7 are adjacent.

**Question 10 [4p.]** Solve the equation

$$2^1 \cdot 2^3 \cdot 2^5 \cdot \ldots \cdot 2^{2x-1} = \frac{1}{2} \cdot 4^{x+8}.$$

**Question 11 [4p.]** For what values of the parameter $p$ the roots $x_1$, $x_2$ of the equation

$$x^2 - (p+3)x + p = 0$$

satisfy the condition $|x_1 - x_2| < 3$?

**Question 12 [5p.]** We draw five numbers consecutively without replacement from the numbers 1 to 20. Find the probability that the second number drawn is divisible by 4 and the last number is divisible by 5.

**Question 13 [5p.]** Consider the function:

$$f : \ [-4,4] \ni x \mapsto |3 - x| - |4 + 2x| + |6x|.$$

(a) Find its smallest and largest value.

(b) Solve the inequality

$$f(x) > 8.$$

**Question 14 [6p.]** In a regular triangular prism, the sine of the angle between the diagonal of the side face and the adjacent side face is equal to $\frac{2\sqrt{3}}{5}$. Calculate the lenght ratio of the edge of the base of the prism to the height of the prism.

**Question 15 [7p.]** The rectangle with edges parallel to the axis $OX$ and $OY$ is inscribed in the figure bounded by the parabola $y = \frac{1}{8}x^2$ and the straight line $y = 6$. Find the coordinates of the rectangle's vertices with the maximal area.