SELECTED EXAM QUESTIONS
AGH University of Krakow
2025

Question 1 [1p.] The median number of days in the months of a non-leap year is equal to

A. 30 **B.** $\frac{365}{12}$ **C.** $30\frac{1}{2}$ **D.** 31

Question 2 [1p.] Let $g(x) = 2x(1-x)$, $h(x) = \frac{x(3x-4)}{2}$. The set of solutions of the inequality $\frac{g'(x)}{h'(x)} \leqslant 0$ is of the form

A. $\left(-\infty, \frac{1}{2}\right] \cup \left(\frac{2}{3}, +\infty\right)$ **B.** $\left[\frac{1}{2}, \frac{2}{3}\right)$ **C.** $\left(-\infty, \frac{1}{2}\right] \cup \left[\frac{2}{3}, +\infty\right)$ **D.** $\left[\frac{1}{2}, \frac{2}{3}\right]$

Question 3 [1p.] The limit

$$\lim_{n\to+\infty} \frac{11+7+3+\ldots+(15-4n)}{n^2+5}$$

is equal to

A. -2 **B.** $+\infty$ **C.** 0 **D.** -4

Question 4 [1p.] The number $\log_{\frac{1}{\sqrt{2}}} \sqrt[3]{2\sqrt{2}}$ equals

A. $-\frac{3}{2}$ **B.** -1. **C.** $-\frac{1}{2}$ **D.** $\frac{1}{2}$.

Question 5 [2p.] Terms of an arithmetic sequence $(a_n)_{n=1}^{\infty}$ satisfy conditions: $a_1 = -10$ and $S_{10} = a_{14} + a_{17}$, where S_{10} is the sum of the first ten terms of the sequence (a_n). Find the common difference of the sequence. In your answer, provide the second, third and fourth terms of the sequence in order.

ANSWER:

Question 6 [3p.] Prove that for every real number b and every positive real number a the inequality $6ab(a-b) \leqslant 3ab^2 + a^3$ holds.

Question 7 [3p.] For what values of parameters a and b is the polynomial $W(x) = 3x^3 + ax^2 + bx - 4$ divisible by $x^2 - 1$?

Question 8 [3p.] Find the equation of the tangent to the curve with equation

$$f(x) = x^2 + \frac{1}{2x-1}$$

at the point with the abscissa $x = 1$.

Question 9 [4p.] A line parallel to the base of a triangle divides the triangle into two figures of equal area. In what ratio does this line divide the sides of the triangle?

Question 10 [4p.] Solve the inequality

$$\sin x + \cos x < \frac{1}{\cos x}$$

for $x \in \langle \pi, 2\pi \rangle$.

Question 11 [4p.] A random experiment consists of tossing a fair coin six times. Calculate the probability of the event that we get at least one tail, given that the first two tosses are both heads.

Question 12 [5p.] On a cylinder with a base radius and height equal to 2, a sphere of radius 1 is placed, tangent to it at the center of symmetry of the cylinder's upper base, thus creating the figure F.

Determine the volume of the cone circumscribed about F, i.e. tangent to the cylinder's upper base and to the sphere, and whose base contains the cylinder's lower base. Make an appropriate drawing.

Question 13 [5p.] Consider the circles O_1 and O_2 with equations

$$O_1 : x^2 + (y-2)^2 = 10, \qquad O_2 : (x+4)^2 + (y-6)^2 = 26.$$

a) Find the intersection points A and B of the circles.

b) Prove that the area of the quadrilateral with vertices at points A, B and the centres of the circles O_1, O_2 is equal to half of the square of the distance between the centres of those circles.

Question 14 [6p.] A sphere with a given radius R and a cone have equal volumes. The lateral surface area of the cone is three times the area of its base. Calculate the height of the cone.

Question 15 [7p.] The sum of all edge lengths of a right triangular prism equals k. Determine the lengths of the base sides and the height of this prism that maximize its volume.